

1: N_1 $N_{S1} - N_1$
 occupanti stati vuoti

2: N_2 $N_{S2} - N_2$

Eq: $N_1 W_{12} (N_{S2} - N_2) = N_2 W_{21} (N_{S1} - N_1)$

(1) $\frac{W_{12}}{W_{21}} = \frac{N_2 (N_{S1} - N_1)}{N_1 (N_{S2} - N_2)} = \frac{\frac{N_{S1} - N_1}{N_1}}{\frac{N_{S2} - N_2}{N_2}}$

$\binom{N_S}{N} = \frac{N_S!}{(N_S - N)! N!}$

$F = E - TS \quad S = k \ln \binom{N_S}{N}$

$F_1 = E_1 N_1 - TS_1 = E_1 N_1 - kT \ln \binom{N_{S1}}{N_1}$

$F_2 = E_2 N_2 - TS_2 = E_2 N_2 - kT \ln \binom{N_{S2}}{N_2}$

$F = F_1 + F_2$

$\frac{\delta F}{\delta N_1} = 0 ; \frac{\delta F}{\delta N_2} = 0$

$\frac{\delta F}{\delta N_1} = \frac{\delta F_1}{\delta N_1} = E_1 - kT \frac{\delta}{\delta N_1} \ln \binom{N_{S1}}{N_1}$

$\ln \binom{N_{S1}}{N_1} = \ln \left[\frac{N_{S1}!}{(N_{S1} - N_1)! N_1!} \right] =$
 $= \ln(N_{S1}!) - \ln[(N_{S1} - N_1)!] - \ln(N_1!)$

STIRLING $\ln(x!) \approx x \ln x - x$
 $= \ln(N_{S1}!) - (N_{S1} - N_1) \ln(N_{S1} - N_1) + (N_{S1} - N_1) +$
 $- N_1 \ln N_1 + N_1 = A_{11}$

$\frac{\delta A_{11}}{\delta N_1} = \left[\ln(N_{S1} - N_1) - \frac{(N_{S1} - N_1)}{N_{S1} - N_1} (-1) - 1 + \right.$
 $\left. - \ln N_1 - \frac{N_1}{N_1} + 1 \right] =$
 $= [\ln(N_{S1} - N_1) - \ln N_1]$

$\frac{\delta F}{\delta N_1} = E_1 - kT [\ln(N_{S1} - N_1) - \ln N_1] = 0$

$\frac{\delta F}{\delta N_2} = E_2 - kT [\ln(N_{S2} - N_2) - \ln N_2] = 0$

$kT \ln \left[\frac{(N_{S1} - N_1)}{N_1} \right] = E_1$

$\frac{N_{S1} - N_1}{N_1} = \exp\left(\frac{E_1}{kT}\right)$

$\frac{N_{S2} - N_2}{N_2} = \exp\left(\frac{E_2}{kT}\right)$

$\frac{W_{12}}{W_{21}} = \frac{\exp\left(\frac{E_1}{kT}\right)}{\exp\left(\frac{E_2}{kT}\right)}$

M $\begin{matrix} i- \\ j \end{matrix}$ $\begin{matrix} f_i \\ f_j \\ 1-f_i \\ 1-f_j \end{matrix}$

$$M f_i W_{ij} M (1 - f_j) = M f_j W_{ji} M (1 - f_i)$$

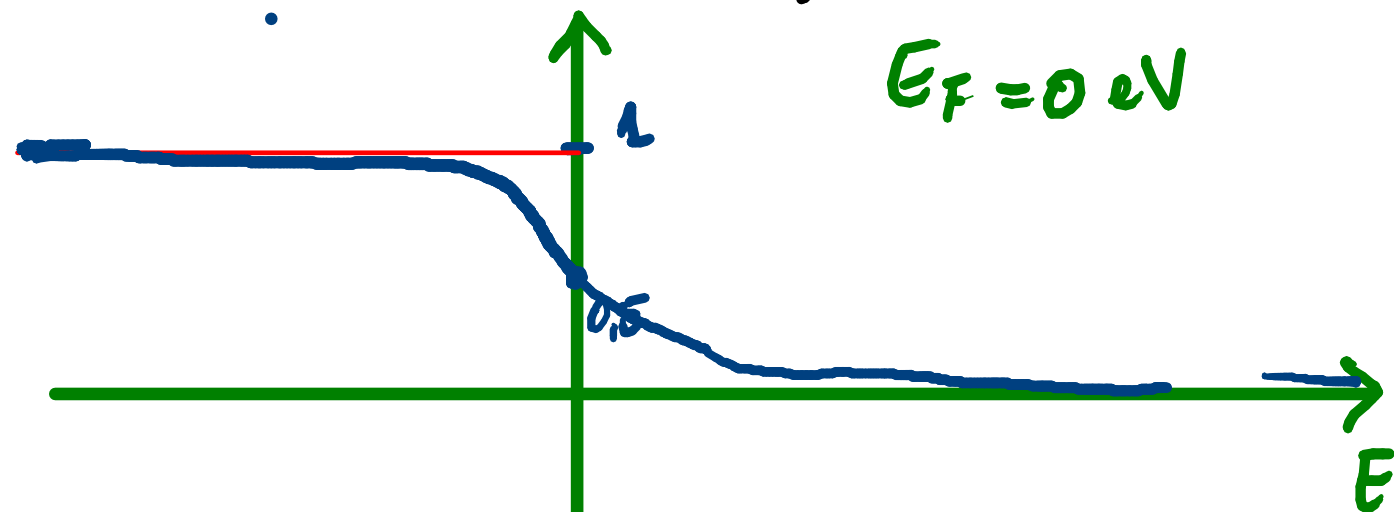
$$\frac{f_i}{1 - f_i} W_{ij} = \frac{f_j}{1 - f_j} W_{ji}$$

$$\frac{f_i}{1 - f_i} \exp\left(\frac{E_i}{kT}\right) = \frac{f_j}{1 - f_j} \exp\left(\frac{E_j}{kT}\right) = C$$

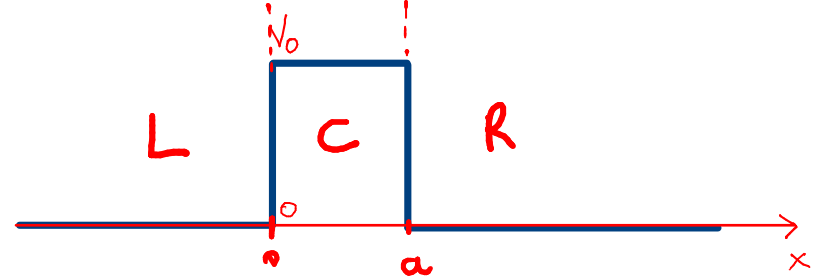
$$f_i = \frac{C}{C + \exp\left(\frac{E_i}{kT}\right)} = \frac{1}{1 + \frac{1}{C} \exp\left(\frac{E_i}{kT}\right)}$$

$$f(E_i=0) = \frac{1}{2} \Rightarrow C = \exp\left(\frac{E_F}{kT}\right)$$

$$f = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$



$$E - E_F \gg kT \quad f \rightarrow \exp\left(-\frac{E - E_F}{kT}\right)$$



$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi(x) = E \psi(x)$$

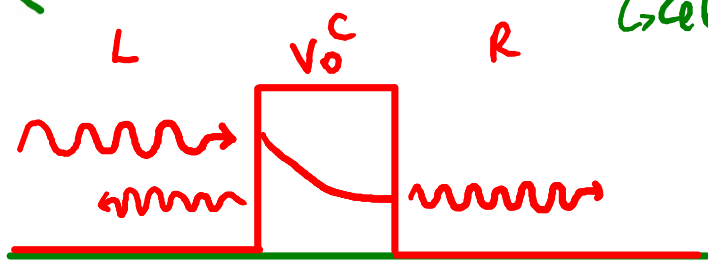
$$\begin{cases} \psi_L(x) = A_1 e^{ik_0 x} + A_2 e^{-ik_0 x} & : x \leq 0 \\ \psi_C(x) = B_1 e^{ik_1 x} + B_2 e^{-ik_1 x} & : 0 < x < a \\ \psi_R(x) = C_1 e^{ik_0 x} + C_2 e^{-ik_0 x} & : x > a \end{cases}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow \begin{cases} k_0 = \frac{\sqrt{2mE}}{\hbar} \\ k_1 = \frac{\sqrt{2m(E-V_0)}}{\hbar} \end{cases}$$

ψ e $\frac{\delta \psi}{\delta x}$ devono essere continui in $x=0$ e $x=a$

$$\begin{cases} \psi_L(0) = \psi_C(0) \\ \psi_C(a) = \psi_R(a) \\ \left. \begin{aligned} \frac{\delta \psi_L}{\delta x} \Big|_{x=0} &= \frac{\delta \psi_C}{\delta x} \Big|_{x=0} \\ \frac{\delta \psi_C}{\delta x} \Big|_{x=a} &= \frac{\delta \psi_R}{\delta x} \Big|_{x=a} \end{aligned} \right\} \begin{cases} A_1 + A_2 = B_1 + B_2 \\ B_1 e^{ik_1 a} + B_2 e^{-ik_1 a} = C_1 e^{ik_0 a} + C_2 e^{-ik_0 a} \end{cases}$$

$$\begin{cases} A_1 + A_2 = B_1 + B_2 \\ B_1 e^{ik_1 a} + B_2 e^{-ik_1 a} = C_1 e^{ik_0 a} + C_2 e^{-ik_0 a} \\ k_0(A_1 - B_1) = k_1(B_2 - B_1) \\ k_1(B_1 e^{ik_1 a} - B_2 e^{-ik_1 a}) = k_0(C_1 e^{ik_0 a} - C_2 e^{-ik_0 a}) \end{cases}$$



$$A_1 = 1 ; A_2 = r$$

$$C_1 = 0 ; C_2 = t$$

$$t = \frac{2k_0 k_1 e^{-ja(k_0 - k_1)}}{(k_0 + k_1)^2 - e^{2ja k_1} (k_0 - k_1)^2}$$

$$r + t = 1$$

