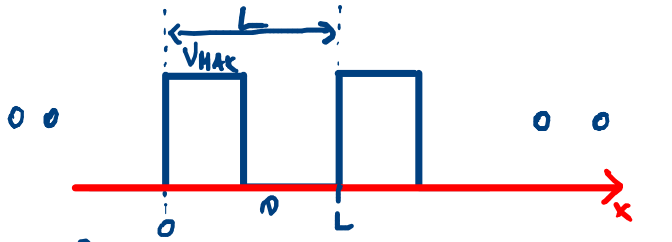
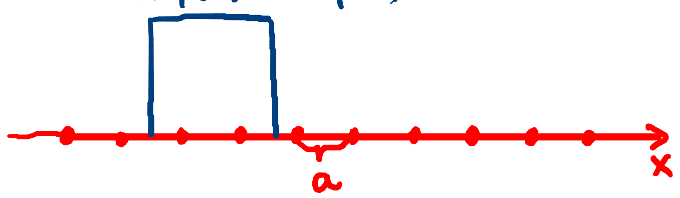


$$\begin{cases} -\dfrac{\hbar^2}{2m_0}\dfrac{d^2}{dx^2}\varphi(x) + U(x)\varphi(x) = E\varphi(x) \\ \varphi(L) = \varphi(0)\,e^{jkL} \quad \underline{\text{Bloch}} \end{cases}$$

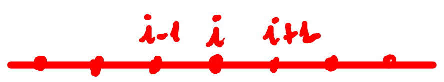
$$H = -\frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial x^2} + U(x)$$

$$H\varphi(x) = E\varphi(x)$$



$$\varphi(x)\Big|_{x=ia} \to \varphi_i \qquad \frac{\partial\varphi}{\partial x}\Big|_{x=ia} = \frac{\varphi_{i+1}-\varphi_i}{a}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\varphi_{i+1}+\varphi_{i-1}-2\varphi_i}{a^2}$$



$$-\frac{\hbar^2}{2m_0}\left[\frac{\varphi_{i+1}+\varphi_{i-1}-2\varphi_i}{a^2}\right] + U_i\varphi_i = E\varphi_i$$

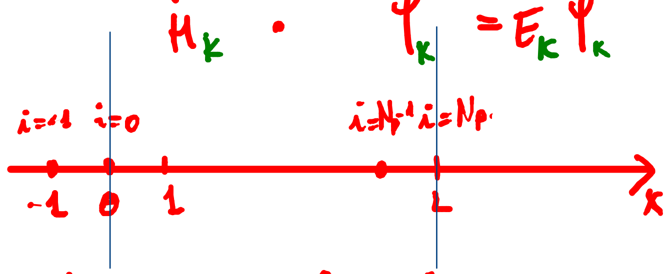
$$t = -\frac{\hbar^2}{2m_0 a^2} \qquad \text{Parametro di Hopping}$$

$$t\varphi_{i+1} + t\varphi_{i-1} - 2t\varphi_i + U_i\varphi_i = E\varphi_i$$

$$t\varphi_{i+1} + (U_i-2t)\varphi_i + t\varphi_{i-1} = E\varphi_i$$

$$\underbrace{\begin{bmatrix} U_0-2t & t & & te^{-jkL} \\ t & U_1-2t & t & \\ & \ddots & \ddots & \ddots \\ te^{jkL} & & t & U_{N_p-1}-2t \end{bmatrix}}_{H_k} \cdot \underbrace{\begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \vdots \\ \varphi_{N_p-1} \end{bmatrix}}_{\varphi_k} = E \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \vdots \\ \varphi_{N_p-1} \end{bmatrix}$$

$$H_k \cdot \varphi_k = E_k\varphi_k$$



$$t\varphi_{-1} + (U_0-2t)\varphi_0 + t\varphi_1 = E\varphi_0$$

$$\varphi_{N_p-1} = \varphi_{-1}e^{jkL} \qquad \varphi_{-1} = \varphi_{N_p-1}e^{-jkL}$$

$$\boxed{te^{-jkL}}\,\varphi_{N_p-1} + (U_0-2t)\varphi_0 + t\varphi_1 = E\varphi_0$$

$$+\frac{\hbar^2}{2m_0}\frac{\delta^2\varphi}{\delta x^2} = [U(x)-E]\varphi$$

$U(x)$ è periodico di periodo L

$$U(x) = \sum_{m=-\infty}^{+\infty} U_m e^{-jm\frac{2\pi}{L}x}$$

$E(k)$ è periodica di periodo $\frac{2\pi}{L}$

$$E(k) = \sum_{\ell=-\infty}^{+\infty} E_\ell e^{-j\ell L k}$$

$E(k)$ sia nota, e che φ_k sia la sua autofunzione relativa

$$E\left(j\frac{d}{dx}\right)\varphi_k(x) = E(k)\varphi_k(x)$$

$$E\left(j\frac{d}{dx}\right)\varphi_k(x) = \sum_{\ell=-\infty}^{+\infty} E_\ell e^{\ell L\frac{d}{dx}}\varphi_k(x) =$$

$$= \sum_{\ell=-\infty}^{+\infty} E_\ell \left[1 + \ell L\frac{\delta}{\delta x} + \frac{1}{2}\left(\ell L\frac{\delta}{\delta x}\right)^2 + \dots\dots\right]\varphi_k(x) =$$

$$= \sum_{\ell=-\infty}^{+\infty} E_\ell \underbrace{\left\{\varphi_k + \ell L\frac{\delta}{\delta x}\varphi_k(x) + \frac{1}{2}\left[(\ell L)^2\frac{\delta^2\varphi_k}{\delta x^2}\right] + \dots\right\}}_{\varphi_k(x+\ell L)} =$$

$$= \sum_{\ell=-\infty}^{+\infty} E_\ell\, \varphi_k(x+\ell L) =$$

$$\varphi_k(x+\ell L) = \exp(-j\ell L k)\varphi_k(x)$$

$$\Rightarrow E\left(j\frac{d}{dx}\right)\varphi_k(x) = \underbrace{\sum_{\ell=-\infty}^{+\infty} E_\ell e^{-j\ell L k}}_{E(k)}\varphi_k(x) = E(k)\varphi_k(x)$$

$$\underbrace{\left[-\frac{\hbar^2}{2m_0}\frac{\delta^2}{\delta x^2} + U(x)\right]}_{E\left(j\frac{\delta}{\delta x}\right)}\varphi_k = E(k)\varphi_k$$

$$E\left(j\frac{\delta}{\delta x}\right) = U(x) - \frac{\hbar^2}{2m_0}\frac{d^2}{dx^2}$$

$$\left[-\frac{\hbar^2}{2m_0}\frac{\delta^2}{\delta x^2} + U(x) + U_x(x)\right]\varphi = E_1\varphi$$

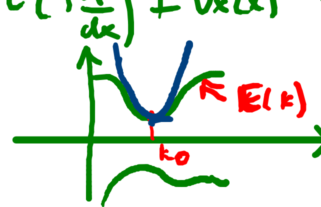
$$\varphi(x) = \sum_k a_k\varphi_k(x)$$

$$\sum_k a_k\left[-\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2} + U(x) + U_x(x)\right]\varphi_k(x) = E_1\varphi$$

$$\Rightarrow \sum_k a_k\left[E\left(j\frac{d}{dx}\right) + U_x(x) - E_1\right]\varphi_k = 0$$

$$\left[E\left(j\frac{d}{dx}\right) + U_x(x) - E_1\right]\sum_k a_k\varphi_k(x) =$$

$$\Rightarrow \left[E\left(j\frac{d}{dx}\right) + U_x(x) - E_1\right]\varphi = 0 \qquad m^* = \hbar^2\left[\frac{\delta^2 E(k)}{\delta k^2}\right]^{-1}_{k=k_0}$$



$$E(k)\Big|_{k=k_0} = E(k_0) + \frac{\hbar^2(k-k_0)^2}{2m^*} + O\left[(k-k_0)^3\right]$$

$$E\left(i\frac{d}{dx}\right) \simeq -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + E(k_0)$$

$$\frac{\hbar^2}{2m^*}\frac{d^2\varphi}{dx^2} = \left[U_x(x) - E_1 + E(k_0)\right]\varphi$$

$$-\frac{\hbar^2}{2m^*}\frac{\delta^2\varphi}{dx^2} + U_x(x)\varphi + \underbrace{E(k_0)}_{E_c}\varphi = E_1\varphi$$